DOCUMENT RESUME

ED 160 382 SE 024 894

TITLE	Experimental Teaching Unit: Fifth Grade Mathematics.
INSTITUTION	Far West Lab. for Educational Research and Development, San Francisco, Calif.
PUB DATE	74
NOTE	64p.; For related document, see SE 024 893; Contains marginal legibility in Tables and Graphs
AVAILABLE FROM	Far West Laboratory for Educational Research & Development, Teacher Education Div., 1855 Folsom Street, San Francisco, California 94103 (no price quoted)
EDRS PRICE	MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS	Elementary Education; *Elementary School Mathematics; Elementary School Teachers; *Instructional Materials; *Learning; *Probability; *Teaching Guides; Units of Study

ABSTRACT

This unit deals with the concept of probability, the use of probability to make predictions, and procedures to gather and display probability data using graphs. Fractions are used to express probabilities. Instructional objectives for probability and for graphing (circle, bar, and line segment graphs) are contained in the booklet as well as suggested activities for attaining these objectives. Answer sheets are included. (Author/MP)

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Introduction to the Unit

This Experimental Teaching Unit is intended to give students some practice in important mathematics skills. There are two sections to the unit, each of which deals with slightly different skills.

For each section the following things are included in this packet:

(1) a brief overview of the skill area
(2) a list of performance objectives for the students
(3) a set of sample instructional materials (you may use those included, select some from another source, or create your own)
(4) some suggestions on ways to use the materials and or other related activities.

Your students will be both pre- and post-tested on many of the objectives listed in this unit. To keep the testing to a minimum not all of the objectives are tested directly. Individual scores on the pretest will be provided for you.

Please spend about 40 minutes each day on work related to these objectives. This means that a student working on unit activities should spend no more than about 40 minutes total during the day.

There are many different ways of organizing the materials and of relating one objective to another. Many exercises can be planned to cover more than one objective. If you wish, objectives from different areas can be included in the same lesson. We are interested in the variety of ways to teach these mathematics skills. You may teach as much of the unit to as many of your students as you wish, and in any manner you choose.

As much as possible, please keep track of what materials and activities you use to teach the unit. To help you do this, please follow these procedures:

(1) Each of the two sections in the ETU is composed of a set of instructional objectives.
  (a) For each section, select the objectives you plan to use.
  (b) Arrange the objectives in the order in which you plan to use them and place the appropriate number in the space provided. (You will use this number to identify each objective on your teaching record, so do this before you begin the unit.)
(2) Use the Daily Teaching Record, to keep track of the instructional objectives and the materials or activities you focus on each day.

At the end of the unit we would like to get information from you on questions like:

(1) What materials did you use (either from the packet or of your own devising)?
(2) In what order were objectives and materials used?
(3) Which materials/activities worked well and which less well?
(4) What was the reaction of the children to the unit?
(5) What procedures did you use for teaching different objectives?

A questionnaire will be provided. We hope you, as teachers, will feel free to try out different approaches to teaching the unit.

OVERVIEW: About Probability

I.

Consider a checker that has a star on one side and a circle on the other:

If we flip the checker in the air once and let it fall on a table, will the star be up or will the circle be up? Clearly, there are only two possibilities: either the star will be up or the circle will be up. (We assume throughout that the checker does not land on its edge, and that the object used is perfectly balanced, or "honest.")

Since only one of two possible results can be obtained by flipping a checker once (a star or a circle), we can express this by the ratio 1 out of 2, or 1/2. This ratio states that there is one chance out of two possibilities that the star will come up (or, equally likely, one chance out of two possibilities that the circle will come up). The ratio 1/2 is an expression of the probability that a star (or a circle) will be obtained by flipping the checker once.

Obtaining a star is one "event," and obtaining a circle is another "event." In one toss of the checker, only one of these two events can occur, and the two possible events are equally probable.

On one toss:

1. The probability of obtaining a star is 1/2 (one chance out of two possibilities).

2. The probability of obtaining a circle is 1/2 (one chance out of two possibilities).

Note that 1/2 also means one-half of the total events.

Probability, then is a numerical measure of the chance that a particular event will occur, compared with the total number of events that could possibly occur.

II.

We use probability in making predictions in our everyday lives. Probability is a concept inherent in such statements as those below. Notice that ratios like those described above as still understood in these statement (e.g. 60% might mean "6 out of 10").

a. "There is a 60% possibility of rain today."
b. "Seven out of 10 school children enjoy drinking milk."
c. "Nearly as much ice-cream is sold in winter as during the summer months.
d. "All the people in the town want an hourly bus service."

e. "Super cars travel 46 miles for every gallon of petrol."
f. "Mrs. Jones says 'Tough' staircarpet is the best. She has had 'Tough' carpet on her stairs for 20 years."
g. "Two out of 3 mothers feed their babies on Growup."
h. "'Floorsmear' makes you floor twice as shiny as any other polish."
i. "'Chloro' washes whiter."

From such statements as these, lively discussions can take place, and questions such as the following can be asked:

-- Is it possible to check the statement? If so, how can I do this?

-- If it is not, should we reject it as meaningless?

-- If it is, how can we verify it?

-- Is the information adequate and sound enough for the general truth of the statement to be accepted?

When we examine the nine statements closely, we can raise all sorts of questions about their validity. For instance,

-- In (a), how often has the weatherman's predictions been correct? On what is he basing his prediction?

-- In (b), would information from all the schools in the area give a result which is likely to be more accurate than from one school only?

-- In (f), children may ask such questions as:

'Has Mrs. Jones a family' If so, how many children were there in the family? Children wear out carpets! True or false?

Has Mrs. Jones ever tried any staircarpet other than 'Tough'? She says 'Tough' is the best!

And even: Is there really a Mrs. Jones, or has she been invented?

-- In (h), 'When do you know the floor is twice as shiny?'

-- (I) can be discussed and dismissed immediately as meaningless.

Statistics can be misleading, as evidenced in the 9 statements above. We are confronted with such statements in advertising in all media, and from such statements we learn to strengthen our own arguments. It is often that we hear:

-- "All the kids have one. Why can't I?"

-- "Children can't read today was well as they could 10 years ago."

-- "Children are worse than they've ever been."

-- "This room is always noisy."

-- "Tommy doesn't know how to behave."

-- "All the children do on the playground is fight."

-- "Parents today are too permissive."

Statistic is one of the branches of mathematics that has developed rapidly in recent years, and, in the future, children will be faced with an ever increasing mass of numerical information. To make intelligent decisions, they will need to know something about inquiring into data that is presented to them as fact (How was the data collected? What are the various types of biases that could have been introduced?)

One way to do this is through the study of probability and the use of some of the simpler statistical operations. Such study will lead to the ability of students to use such terms to describe an event or a phenomena as:

-- <u>certain</u>: "It will rain today because I see rain falling outside."

-- <u>uncertain</u>: "It is cloudy outside, but I cannot say that it will rain today." (i.e., "I need more information before I am willing to make a prediction.")

-- <u>impossible</u>: "The temperature is 95°, it is impossible for it to snow here."

III.

There are other things about probability that we must know before we can properly interpret data.

For instance, returning to the example of the two checkers in Section I of this discussion, the probability of a particular event occurring is the ratio of the number of ways the particular event can occur to the number of possible events. We can express the ratio mathematically as indicated on the next page:

$$\text{Probability of a particular event occurring} = \frac{\text{Number of ways the event can occur}}{\text{Total number of possible events}}$$

In flipping a checker once, the total number of possible events is 2. Since the probability of obtaining a star is 1/2 and the probability of obtaining a circle is 1/2, what else can we say about the nature of probability from our experiment? The sum of the probabilities of all the possible events is $\frac{1}{2} + \frac{1}{2}$, or $\frac{2}{2}$, or 1. This agrees with our common sense: if we flip a checker, it must result in one of the possible events (since we have barred the possibility of the checker landing on edge).

To illustrate another aspect of this situation, imagine that you have a checker with a star on both sides. What is the probability of obtaining a circle by flipping the checker?

$$\text{Probability of a particular event occurring} = \frac{\text{Number of ways the event can occur}}{\text{Total number of possible events}} = \frac{0}{2} = 0$$

In short, the event cannot happen.

We can also obtain theoretical data on probability by computation. For example, we have said that the probability of obtaining a star by flipping a checker once is $\frac{1}{2}$ (or 1 out of 2). Suppose we flip the checker twice in succession. What are the possible outcomes? Four results are possible, as shown in the table below.

FIRST TOSS	SECOND TOSS	POSSIBLE RESULTS
A. S	S	Star on first toss Star on second toss
B. C	S	Circle on first toss Star on second toss
C. S	C	Star on first toss Circle on second toss
D. C	C	Circle on first toss Circle on second toss

The table shows that the probability of getting a star and a circle is greater than that of getting two stars or two circles.

-- What are the chances that flipping one checker two times will turn up a star both times? One out of 4 chances, or $\frac{1}{4}$.

-- What are the chances that the two tosses will turn up a circle both times? One out of four chances, or $\frac{1}{4}$.

-- What are the chances that the two tosses will turn up one star and one circle? Two out of four chances, or $\frac{2}{4}$.

Three different combinations can be obtained by tossing one checker twice. The combinations are:

(1) two stars (Row A);
(2) two circles (Row D);
(3) one star and one circle (Rows B and C).

By performing such experiments, we can see whether our theoretical data hold true. However, all experimental data on probability are based on a "long run," that is, a large number of trials. The greater the number of trials we make (the longer the run), the more likely it is that experimental data will approximate our theoretical data. For example, the familiar expression "fifty-fifty chance" simply means that out of 100 trials (for example, flipping a checker 100 times), the number of stars (or circles) we are likely to flip will be closer to 50 than to 100. However, this is one of the most misunderstood points about probability. It could happen (though it is unlikely) that a checker would register 99 stars consecutively. What would be the probability of obtaining a star on the next toss? Still one out of two. The probability of obtaining a star is one out of two on each flip. But even if we did obtain 99 stars and one circle out of 100 tosses, it is likely that in a run of 1000 trials, we would still come closer to 500 stars and 500 circles.

We have examined the possibilities resulting from tossing a checker twice. Note that tossing one checker twice in a row is the same as tossing two checkers at one time. In both situations the same combinations occur.

PROBABILITY

The objectives for this part of the ETU are:

_____ Given a probability problem, the student will understand that the chance of an event happening equals the total number of successes divided by the total number of tries possible.

Example:

One way of stating this is "1 out of 3" to explain that, when Jane was asked to pick the blue ball out of a sack containing one blue ball and two red balls, her chance of doing so successfully was "1 out of 3". This is stated arithmetically as each chance being 1/3, of an opportunity to correctly do so, or $\frac{1}{3} = \frac{\text{the \# of successful events}}{\text{the total possible tries.}}$

_____ Given a set of events the student will express as a ratio such as four out of six, the probability that the particular event may occur.

_____ Given a set of events the student will express in fractional form the probability that the particular event may occur.

_____ Given a probability situation a student will be able to select from the following words the word that best describes the situation:

likely
more likely
equal chances
certain
uncertain
impossible

_____ Given the probability of a particular event occurring, the student will be able to provide the probability that it will not occur.

Example:

If you roll a die, the probability of getting a 4 is 1/6, or 1 out of 6. The probability of not getting a 4 (i.e. getting anything but 4) is 5/6, or 5 out of 6.

_____ Given an experimental situation in which an event may or may may not happen, the student will understand that the probability of a thing happening is always at least zero and never more than 1.

Example:

The probability (or chance) of getting a "7" on a roll of a single die is 0 since there is no way to roll a "7". This is an impossible event.

The probability of getting one of the numbers 1, 2, 3, 4, 5, or 6 is 1 since $\frac{6}{6} = 1$. This is a certain event.

The probability of getting a "3" on a roll of a single die is $\frac{1}{6}$ (one way out of 6 possibilities). This is an uncertain but possible event.

_____ Given a specific event a student will predict that the probability will be certain, uncertain, or impossible.

_____ Given words that describe a probability situation (certain, uncertain, impossible), the student will choose events that fit the descriptions.

_____ Given a probability statement, the student will determine the possible number of outcomes.

_____ Given a probability statement, the student will illustrate all possible outcomes using a variety of displays, e. g., tree diagram, list, graph, chart.

_____ Given an illustration of a probability statement, the student will provide probability of a specific event happening.

_____ Given an illustration of a probability statement the student will provide probability of all possible outcomes.

_____ Given a probability situation in which two things may happen the student will find probability that both will occur together.

_____ Given a probability situation, the student will determine the probability of two events happening by listing all possible happenings.

Example:

Suppose there are 2 chocolates and 1 butterscotch candy in a bag. Suppose also that there are 3 pieces of Double Bubble gum and 2 pieces of Bazooka bubble gum in a box. If a person draws one piece of candy from the bag and one piece of bubble gum from the box the probability of drawing a chocolate and a piece of Double Bubble is:

$$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$$

since there is a 2/3 (2 out of 3) chance of drawing a chocolate and a 3/5 (3 out of 5) chance of drawing Double Bubble.

GRAPHING

Opportunities to develop graphs and charts will arise as students collect data that needs to be organized into a form that can be easily communicated. In this way, students will also develop skill in interpreting such graphs and charts as an important part of their reading in social studies, science, current events, etc.

Three forms are presented in this section:

(a) the circle graph, used to illustrate the parts that make up a whole;

(b) the bar graph, used to illustrate comparisons among similar events or objects; and

(c) the line segment graph, used to illustrate the trend (or development) of a single phenomena, event, or object.

The objectives for this part of the ETU are:

_____ For a given set of data (e.g. graph, list, etc.) the student can interpret that data by giving appropriate information.

_____ For a specific situation the student will gather appropriate data by accurate observation and by keeping records, i.e., listing, tallying and categorizing, etc.

_____ For a specific data that has been collected, the student will display the data by listing, diagramming, graphing, etc.

_____ Given data which has been gathered, recorded and displayed in an appropriate form, the student will interpret it by using "mean", "median", "mode", and "range".

_____ Given data which has been gathered, recorded, and displayed in appropriate form, the student will interpret it by naming the appropriate graph.

PROBABILITY - Suggested Activities

101. Have students toss a thumbtack (or any object with only 2 different sides) onto a level hard surface. It can fall pin up (U) or pin down (D). Have the students carry out 50 trials, guess the results first and then record the frequency of each outcome. Discuss the results.

102. To predict the result of flipping a coin, have students flip a coin 10 times and tally each outcome (heads or tails). Encourage students to compare and to combine results with another student. Ask students to predict the outcomes of their next flip based on their previous trials.

103. A box contains six blue marbles and four red marbles. Ask a student the color marble he is most likely to draw. Now blindfold the student and have him choose a marble from the box.

104. Mix up a hat full of marbles and select one marble blindfolded. Now look at it and record its color. Repeat this 100 times and record your results with other members of your group and discuss them.

105. Use three coins to perform these experiments. Toss the three coins fifty times. Record the number of times the coins turn up in each of the four possible ways: 3 heads; 3 tails; 2 heads 1 tail; 1 head 2 tails. Determine the probability of each of the following:
    a. turning up 3 heads
    b. turning up 2 heads and 1 tail
    c. turning up 1 head and 2 tails
    d. turning up 3 tails

106. Toss a die 50 times. List all possible outcomes on a chart and tally the outcomes. Determine the fraction of total trials.

107. Suppose that a biologist injects a vaccine into several rats and then exposes all of them to the same disease. If a rat gets the disease, the biologist records (S) for sick; if not, he records (W) for well. His record is SSWWWW. If another rat is given the vaccine and then exposed to the disease, do you think it will probably stay well? Explain your answer.

108. Roll a pair of dice 50 times. In each roll multiply the two numbers. Guess the product that will appear most frequently. Repeat the experiment. Did the same product occur?

109. Take two boxes. Put 2 red marbles and 3 blue marbles in one. Put 1 white marble and 4 green marbles in the other. Without looking, if you pick one marble from each box, what is the probability that you will choose one red and one green marble? Pick one from each box and record your results.

110. a) If you toss a coin, the probability is 1 out of 2 that the coin will land heads up. What is the probability that it will land tails up?

b) Does the fraction 1/2 represent this probability?

c) Can you write 5 more fractions that represent the same probability as 1/2?

111. Suppose you are blindfolded and a friend places two different pairs of shoes in front of you and tells you to choose one pair. What is the probability that the first two shoes you pick up will be a matched pair?

112. If you were to choose a number at random from the set:

(2,3,4,5,12,13,14,15,22,23,24),

a) what is the probability that the number would be an even number?

b) what is the probability that the number would be an odd number?

c) what is the probability that it would be an even number and a multiple of 5?

d) what is the probability that it would be an odd number and a multiple of 5?

113. Suppose you have 5 red poker chips, 4 blue chips, and 3 white chips in a bag.

a) what is the probability of getting a blue chip on the first draw?

b) what is the probability of getting a white chip on the first draw?

c) what is the probability of not getting a white chip on the first draw?

d) Suppose you got a red chip on the first draw. What is the probability of getting another red chip on the next draw?

e) If you know that the probability of getting a blue chip is zero, what does this tell you about the colors of the chips in the bag?

114. Toss a coin 25 times and keep a record of the number of times heads appears and the number of times tails appears. From your experiment what do you think is the probability of getting heads?

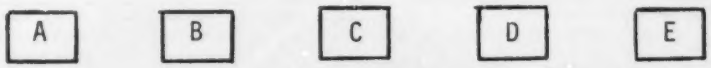
Now toss the coin 50 times and record the results the same way. How do the two charts compare? Do you think there is any change in the probability of the number of times heads will appear?

115. Many games such as Monopoly or parcheesi use two dice with a certain number of dots on each face of each die. Usually the dots number from 1 to 6 on each face.

Toss a die 60 times and record the number of times each of the numerals, 1, 2, 3, 4, 5, or 6 appears.

Can you think of a way to record the results of this kind of experiment using two dice at a time? Try it!

116. Make a set of cards with the letters A, B, C, D, E.



Make 14 "A" cards, 8 "B" cards, 6 "C" cards, 6 "D" cards, and 2 "E" cards.

If you shuffle the cards and draw one card, what is the probability of getting an "A" card?

What is the probability of getting an "E" card?

Which cards would have equal chances?

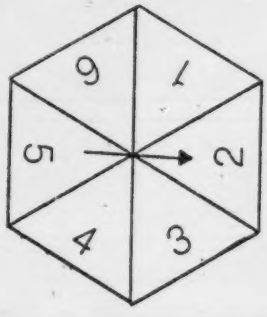
Make a simple chart to record this experiment. Make a guess as to the letter of the card you think you will get. Write it down. Draw a card and write down the letter of the card you actually got. See how many times you win.

117. You have probably played games that have a spinner like the one below. If you do not have one, make one like it with card board and a brass, round head paper fastener. Use stiff flat cardboard and make sure the spinner turns freely.

If you spin the spinner just one time, what is the probability of landing on 6?

What is the probability of landing on 3?

Spin the spinner 20 times and record the results.



Numeral	1	2	3	4	5	6
Occurs						

117. Which numeral occurred the greatest number of times? _____

Which numeral occurred the least number of times? _____

According to your experiment, which numeral seems most likely to occur? _____

Which one seems least likely to occur? _____

Try this experiment two more times and compare the three records you now have. Have you changed your opinion? _____

118. There are many practical uses of probability in science, advertising, insurance, TV ratings, and test writing. See whether you can find information about some practical use of probability and write a report on it, or describe an experiment or project that demonstrates it.

119. Ask children to choose a partner. Each player has 12 counters. Play for these in a game with 2 dice. Whenever 11 is thrown (by either), A gives a counter to B. Whenever 9 is thrown (by either), B gives a counter to A. The player who first wins all the counters wins the game. Which player (A or B) is most likely to win, and why?

120. Ask students to take a die. Throw it until you get a 6. How many throws did you need? Record this. Run a number of trials recording each time. Plot a bar graph of the frequencies of 1, 2, 3, etc. throws. What was the most common number of throws? What was the average number of throws?

121. Have students shuffle a pack of playing cards. Deal out 4. How many are clubs? Record this. Replace the cards, shuffle and try again. Carry out a number of trials. Make a bar graph with the frequencies of no clubs, 1 club, 2 clubs, 3 clubs, or 4 clubs.

122. Have students shuffle a pack of cards. Deal out 4. How many of the 4 are black? Record this. Replace the cards. Run a number of trials, Plot a bar graph showing how many times you had no blacks, 1, 2, 3, or 4 blacks.

123. Take two boxes. Put 3 red and 2 blue marbles in one and 2 yellow and 1 green in the other. If you pick one marble from each box without looking, what are all of the possible outcomes you could get?

124. Give students some dominoes. Have them remove all the dominoes with one or two blanks. 21 should be left. Shuffle them face down on the table. Draw one at random. Count the total number of pips on it. Replace it and reshuffle. Run a number of trials and plot a bar graph showing the frequencies of different totals of pips.

125. Have students take a book. Count the number of letters in each word in one paragraph of the book. Plot a bar graph showing the number of words with one, two, three... etc. letters. Take a very difficult book. Do the same. Compare the two graphs.

GRAPHING - Suggested Activities

401. Role two die from a cup 100 times. Record the sum of the two numbers on a tally sheet. Make a bar graph showing which sum you got the most times. The least. Explain your results. The possible reasons for your results.

402. Survey classmates to determine the birthdays of each member. What are the probabilities of two students having a birthday on the same day of the same month. List the birthdays and illustrate them on a graph.

403. Survey students to determine how they spend the hours of the day. Have students draw a circle graph listing acitivities.

404. Find the mean, median, mode, and range of daily temperatures during the week. List them on a graph.

405. Have several students run the 25-yard dash. Clock the time for each student and record the data on a graph. Find the mean, median, mode, and range.

406. Have your students use the newspaper to list the number of 30-minute TV programs in each of the following categories: sports, news, education programs, quiz or game shows, comedy. Illustrate the listings on a graph. What conclusions can be drawn from the listings?

407. Take a poll of the kinds of pets students have in the class. Graph the results.

408. Have students line up at the basketball net and ask them to throw the ball ten times. Repeat the activity to find the average number of successes. Repeat the activity many times, recording the results and graphing them. Have students interpret results.

409. Have students make a circle graph on ways they spend their allowance during a given week.

410. Using the attendance record, have students graph the frequencies of absenteeism on a chart. Compute the mean, median, mode, and range of the results.

411. Draw a circle graph illustrating your daily diet.

412. Conduct a survey of your classmates favorite sport. Make a circle graph to illustrate this data.

413. Imagine you have the following ingredients: vanilla ice cream, 1 cherry, nuts and chocolate syrup. Make a tree diagram listing all possible combinations.

414. Use the world almanac to show the birth and death rate for five states during a given year. Construct a bar graph to illustrate this data. What conclusions may be drawn from the graph?

415. Draw a bar graph to illustrate the ten top selling records of the year. (Data may be obtained from the world almanac.)

Probability

Probability is an exciting branch of mathematics in which mathematicians use measures of chance to answer questions for science and industry. On this and the next three pages, you will learn something about probability and how to use measures of chance to answer certain kinds of questions.

1. Suppose you tossed a coin in the air. Is it possible that the coin will land "heads"? _____

Is it also possible that the coin will land "tails"? _____ Then the coin can land in one of two possible ways.

One of the two possible outcomes when you toss a coin is heads. We say that the probability of getting heads on one toss is 1 out of 2, or 1/2.

The other one of the two possible outcomes when you toss a coin is tails. Can we say that the probability of getting tails in one toss is also 1 out of 2, or 1/2? _____

2. Toss a coin 50 times and see how many heads and how many tails you get. Use the chart in the next column to record the results of the experiment. Put a mark in the HEADS column each time the coin lands heads. Put a mark in the TAILS column each time the coin lands tails.

3. What is the total number of marks for heads? _____ for tails? _____ The more times you toss a coin, the nearer you are likely to come to the probability 1/2. Toss the coin 50 more times. Then combine the results of the first 50 tosses. What is the total number of marks for heads? _____ What is the total number of marks for tails? _____

TOSS	HEADS	TAILS
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
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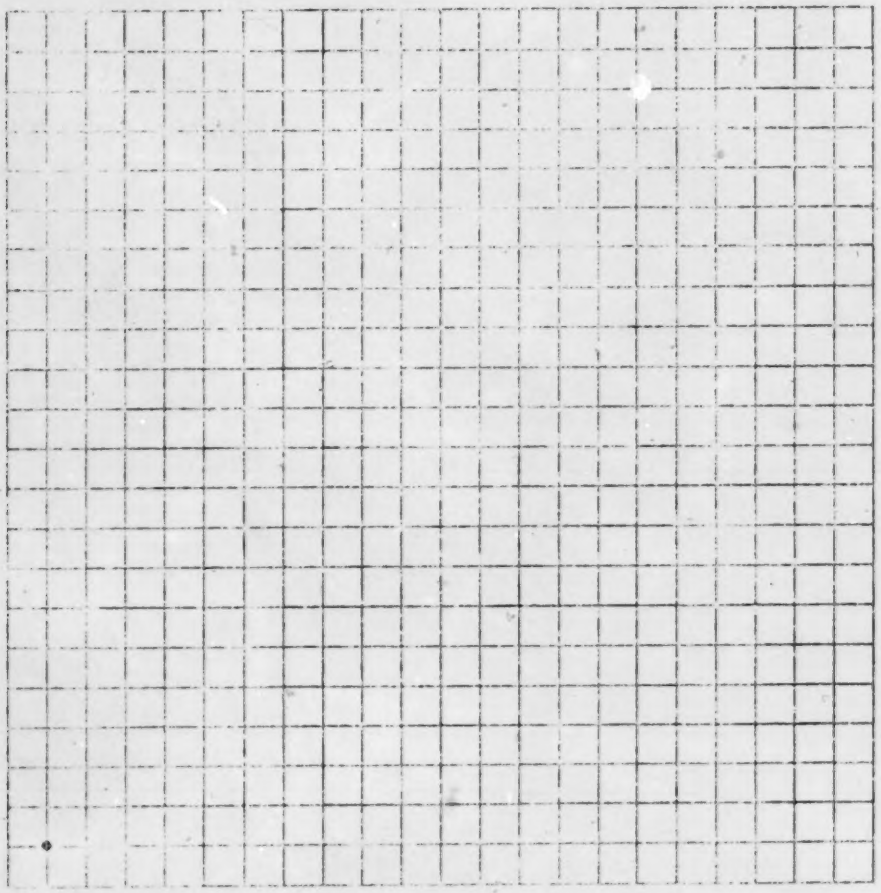
Probability

One of the most interesting applications of probability is a random walk The work random means "unguided" or "left to chance." A good example of a random walk is the movement of molecules of air in a room. They move about in an unguided, or random, way. In the experiment below, the pattern that you make by tossing a coin will be a very simple one. The patterns made by the molecules of air are much more complex.

On the grid below, you are to record the results of tossing a single coin. Toss the coin once. If it lands heads, start at the dot on the paper and draw a line segment one space to the right. If the coin lands tails, start at the dot and draw a line segment one space up toward the top of the paper. Keep tossing the coin and drawing line segments either to the right or up from your last move. If the coin lands heads several times in a row, keep drawing line segments to the right. If the coin lands tails several times in a row, keep drawing line segments up. Toss the coin until the last line segment goes of the grid. See what pattern develops.

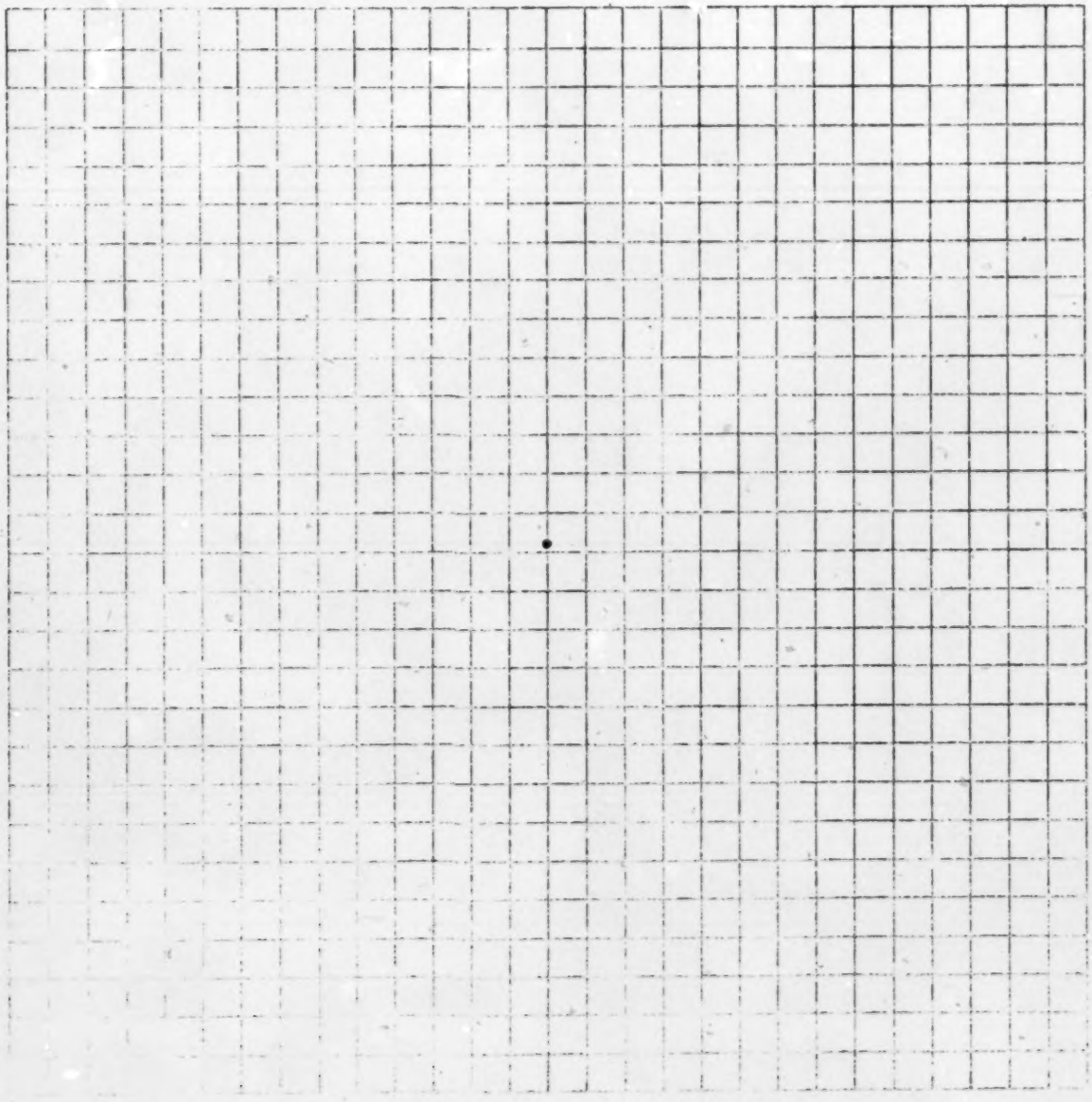
Repeat the experiment and draw the line segments, with a colored pencil or a crayon. Is the pattern different the second time?



Probability

For this random walk you will need a die (one of a pair of dice). On the grid below, start your movements from the dot in the center of the paper. Toss the die and if "1" comes up, draw a line segment one space to the right. If "2" or "4" comes up draw a line segment one space to the left. If "3" or "5" comes up, draw a line segment one space up. If "6" comes up, draw a line segment one space down. Toss the die fifty times and see what pattern develops.

Repeat the experiment and see if an entirely different pattern develops. Then repeat the experiment again. Each time the pattern should be different.



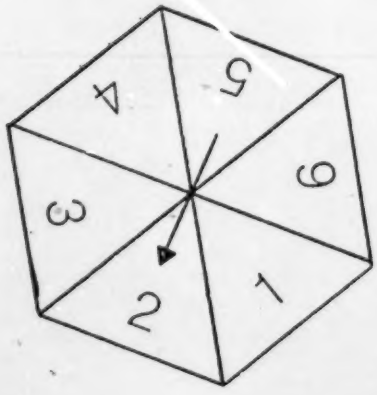
POSSIBLE OUTCOMES

1. How many possible outcomes are there when you toss a penny? _____

2. How many possible outcomes are there when you toss a penny and a nickel? _____

3. How many possible outcomes are there when you spin the pointer on the spinner below? _____

The spinner shown was spun 30 times. The results are recorded in the table below. Complete the table by doing exercies 4, 5, 6, 7, 8, 9, and 10. (Blank and incomplete spaces in table.)



OUTCOME	TALLY	NUMBER FOR EACH OUTCOME	FRACTION OF TOTAL TRIALS
1	𝍸	5	$\frac{5}{30}$
2	𝍸 ||	7	4. $\frac{}{30}$
3	𝍸 |	5.	6. $\frac{}{30}$
4	|||	7.	8. $\frac{}{30}$
5	𝍸 ||||	9.	10. $\frac{}{30}$

DIE-TOSSING EXPERIMENT

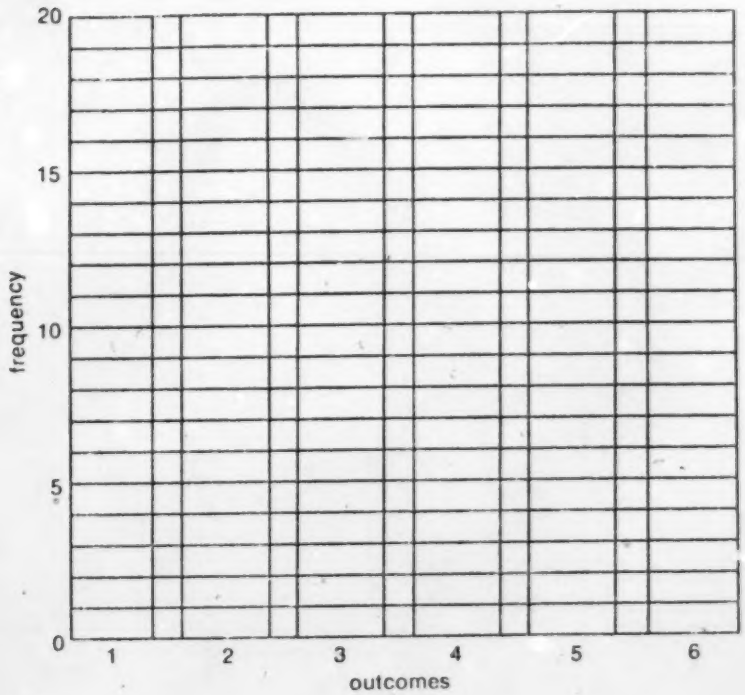
1. In this experiment, a die is tossed. One of two dice is called a die. When a die is tossed, the possible outcomes are 1, 2, 3, 4, 5, and 6. The number of times an outcome occurs in an experiment is called the frequency.

   How often do you think the 5 will turn up? _____

2. Toss a die 30 times. Tally the outcomes in the table below as you toss the die. Then complete the table.

Outcome	Tally	Frequency (number)	Fraction of total trials
1			$\frac{}{30}$
2			
3			
4			
5			
6			
	Total		$\frac{}{30}$

3. Using your results from the table on the previous page, make a bar graph. Draw each bar to show the frequency for an outcome.



4. Combine your results with those of another student. Complete the table below.

Outcome	Your frequency	Partner's frequency	Combined frequency	Fraction of total trials
1				$\frac{\quad}{60}$
2				
3				
4				
5				
6				
TOTAL				$\frac{\quad}{60}$

5. How many outcomes were possible in the die-tossing experiment? _____

6. Was the fraction of total trials for each outcome about $\frac{1}{6}$? _____

When tossing a fair die, each of the 6 numbers has the same chance of appearing. In this case, we say that the 6 numbers are equally likely.

7. What might be the reason if 5 came up nine out of ten times?

8. Are the fractions for total trials in the table above closer to $\frac{1}{6}$ than those in the table on the preceeding page?

The more trials you perform with a fair die, the closer to $\frac{1}{6}$ you would expect the fraction to be.

MEASURES OF CHANCE

1. Before you take a chance, you should know what the risk is. Three coins -- a penny, a nickel, and a dime -- are tossed.

   The tree diagram below shows all the possible outcomes. Use H and T to show the results. Some have been done for you.

2. How many different outcomes for three coins are possible? _____

3. What fraction of these possible outcomes give 3 heads (H,H,H)? _____

4. What fraction of these possible outcomes is each different outcome? ___

5. How many times would you expect to get H H H? _____

6. How many times would you expect to get H H T? _____

THREE COIN EXPERIMENT

1. Find out what outcomes you actually get when you toss 3 coins 32 times. Record your results in the table below.

Outcome			Tally	Frequency	Fraction of total trials	Expected fraction of total trials
Penny	Nickel	Dime				
H	H	H			$\frac{\quad}{32}$	$\frac{1}{8} = \frac{\quad}{32}$
H	H	T			$\frac{\quad}{32}$	$\frac{1}{8} = \frac{\quad}{32}$
H	T	H				$\frac{1}{8} = \frac{\quad}{32}$
H	T	T				$\frac{1}{8} = \frac{\quad}{32}$
T	H	H				$\frac{1}{8} = \frac{\quad}{32}$
T	H	T				$\frac{1}{8} = \frac{\quad}{32}$
T	T	H				$\frac{1}{8} = \frac{\quad}{32}$
T	T	T				$\frac{1}{8} = \frac{\quad}{32}$
			TOTAL	32		

2. Which outcomes resulted in a fraction that is equal to $\frac{1}{8}$ or $\frac{4}{32}$?

3. The fraction of total trials for each outcome in an experiment is the experimental probability for that outcome. The probability of an outcome is a prediction of how often the outcome will occur. To find the probability of an outcome from an experiment, use the fraction

$$\frac{\text{number of times the outcome occurred}}{\text{number of trials}}$$

Circle the fractions of total trials for each of the outcomes that are $\frac{3}{32}$ or $\frac{5}{32}$.

PREDICTING COIN TOSSES

1. To find the probability of an outcome, you first record all the possible outcomes. Then complete the table below for coin tossing.

Number	Possible outcomes for coins tossed	Number of possible outcomes
1 coin	(H) (T)	2
2 coins	(H)(H), (H)(T), (T)(H), (T)(T),	
3 coins	(H)(H)(H), (H)(H)(T), (H)(T)(H), (H)(T)(T), (T)(H)(H), (T)(H)(T), (T)(T)(H), (T)(T)(T)	8

2. Sometimes we assume that the outcomes in an experiment are equally likely. On a fair coin, heads and tails are equally likely. There is 1 way to get heads. There are 2 possible outcomes. The probability for heads is $\frac{1}{2}$ for a fair coin.

   What is the probability of tails for a fair coin? [—]

3. If a fair coin is tossed 20 times, about how many times do you expect to get heads? _____

4. If outcomes are equally likely, the probability of each outcome is

$$\frac{1}{\text{number of possible outcomes}}$$

   When tossing 2 coins, there are 4 different outcomes. One outcome is H,H. What is the probability of getting 2 heads when tossing 2 coins?

   [—]

5. If 2 coins are tossed 20 times, about how many times do you expect to get 2 tails?

6. When tossing 3 coins, there are 8 different outcomes. What is the probability of getting each outcome?

7. If 3 coins are tossed 32 times, about how many times do you expect to get 3 heads? _____

8. The table below shows the probabilities for equally likely outcomes. Complete this table.

Number of possible outcomes	Probability for each outcome
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	
	$\frac{1}{5}$
9	

9. The table below shows the probabilities for coin tosses. Complete this table.

Number of fair coins	Number of possible outcomes	Probability for each outcome	Predicted number of times each outcome occurs in 64 trials
1	2	$\frac{1}{2}$	32
2		$\frac{1}{4}$	
3			
4	16		

10. If you toss coins 64 times, will you always get the number of outcomes predicted in the table above? _____

DICE TOSS

1. If you toss a fair die, about how many times would you predict a 2 would be tossed in 60 trials? _____

2. A die is tossed 60 times. The results are shown in the table below. Complete the table by doing exercies 2, 3, 4, 5, 6, 7, 8, and 9 (blank and incomplete spaces in table).

Outcome	Frequency	Fraction of total trials
1	14	2. $\frac{\quad}{60}$
2	9	3.
3	4.	$\frac{10}{60}$
4	5.	$\frac{12}{60}$
5	6.	$\frac{6}{60}$
6	9	7.
TOTAL	8.	9.

DRAWING CARDS FROM A HAT

Place a King of spades, clubs, diamonds, and hearts in hat and mix them up.

1. How many cards are in the hat? _____

2. How many Kings are in the hat? _____

Draw a card, look at it, and replace it in the hat.

3. If you make 10 draws, what fraction of draws will be Kings?

4. Since all of the cards in the hat are Kings, what is the probability of drawing a King? _____

5. If you make 10 draws from the hat, what fraction of draws will be Queens?

6. What is the probability of drawing a Queen in this experiment? _____

7. Circle the outcome that is certain. Queen King

8. Circle the outcome that is impossible. King Queen

COIN TOSSING

When a coin is tossed, there are two possible ways it can land.

One outcome is:

Heads (H)

The other outcome is:

Tails (T)

1. Toss a coin 20 times. Tally each outcome in the table below as you toss the coin. Complete the table.

Outcome	Tally	Number for each outcome	Fraction of total trials
H			$\frac{\quad}{20}$
T			$\frac{\quad}{20}$
	Total		$\frac{\quad}{20}$

2. Combine your results from the table above with those of another student. Fill in the table below.

Outcome	Your number for each outcome	Partner's number for each outcome	Total number for each outcome	Fraction of total trials
H				$\frac{\quad}{40}$
T				$\frac{\quad}{40}$
Total				$\frac{\quad}{40}$

3. How many different outcomes were possible in this experiment? _____.

We say that the chances that a "fair" coin will land heads is about $\frac{1}{2}$.

4. Was the fraction of the total trials for each outcome in this experiment about $\frac{1}{2}$ (about $\frac{20}{40}$)? _____

5. If a coin landed heads 99 times out of 100, would you think it was fair? _____

6. Jim performed three experiments by tossing a coin. In one experiment, he tossed the coin 20 times. It landed heads 8 times. How many times did it land tails? _____

7. Complete the table below:

Number of tosses	Number of heads	Number of tails
20	8	
15	4	
100	50	

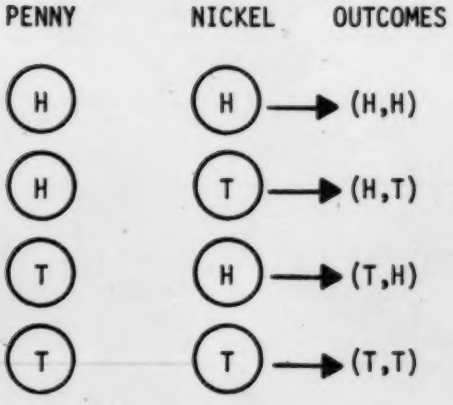
8. Use the results in the table below to find the fraction of total trials for each outcome.

Fraction of heads	Fraction of tails	Sum of the 2 fractions
$\frac{8}{20}$		
$\frac{4}{15}$		
$\frac{50}{100}$		

9. If you reduce the sums of the 2 fractions, what is the sum in each case? _____

10. Two coins, a penny and a nickel were tossed. The outcomes are the different ways the two coins may fall. With a penny and a nickel, there are four possible outcomes.

Toss two coins 20 times. Record your results in the table below.



Outcome: Penny	Outcome: Nickel	Tally	Number for each outcome	Fraction of total trials
H	H			$\frac{\quad}{20}$
H	T			$\frac{\quad}{20}$
T	H			$\frac{\quad}{20}$
T	T			$\frac{\quad}{20}$
		TOTAL		$\frac{\quad}{20}$

11. Your experiment shows that your chance of tossing two heads is about _____ out of 20.

12. Your chance of tossing one head and one tail is $\frac{\quad}{\quad}$

13. Your chance of tossing two tails is $\frac{\quad}{\quad}$

WHAT IS CERTAIN?

When events occur or are going to occur, we have different ways of talking about them. When we are sure something will happen, we say it is a certain event. For example, if you spin a spinner that is all red, getting red is a certain event. Its probability is 1. The probability of getting blue on the same spinner is 0, which we call an impossible event.

On a spinner with more than one color, the probability of getting one of the colors is greater than 0, but less than 1. We say that these are uncertain events.

Consider flipping a coin. There are equal chances of getting a head or a tail. It is likely that you will get one of them, but one is not more likely than the other.

Exercise: Using the words: likely, more likely, equal chances, certain, uncertain, and impossible, identify the following events:

1. Toss one coin. Getting a head. _____
2. Getting a head vs. getting a tail. _____
3. Spin three color spinner. Getting a particular color. _____
4. Spin one color spinner. Getting that color. _____
5. Spin three color spinner. Getting a color not on the spinner. _____
6. Toss one die. Getting a 1, 2, 3, 4, 5, or 6. _____
7. Toss one die. Getting a 7. _____
8. Toss two dice. Getting a sum of 13. _____
9. It is raining. You walk outside without an umbrella. You will get wet. _____
10. You are launching a space ship. It will take off. _____

SUCCESS or FAILURE

Sometimes we consider certain outcomes of an experiment to be "successes" and the other outcomes to be "failures". This does not mean that they are good or bad but just a way to name the different outcomes.

For example, suppose you call getting a head (H) when you toss one coin, a success. Then getting a tail (T) would be called a failure. Notice that the probability of getting a head is 1/2 and the probability of getting a tail is 1 - 1/2 = 1/2. The total of the probabilities is 1. Complete Table 1, by filling in the missing information.

Table 1.

Experiment	Outcomes	Successes	Failures
Toss a die	1,2,3,4,5,or 6	2 or 6	1,3,4or5
Player at bat	gets a hit, does not get a hit	gets a hit	
Toss a coin	H, T	H	
Toss two coins	HH,HT,TH,TT	TT,HH	
Launch a spaceship	takes off,does not take off	takes off	

How did you figure out the "failures"? _____

** Notice that: OUTCOMES - SUCCESSES = FAILURES

Another way to say this is that the "failures" are those outcomes that are not considered "successes".

Notice also that the "successes" are the outcomes not considered "failures". When you toss a single coin one time, not getting a head means getting a tail and vice versa.

Now let's look at the probabilities.

Complete this table.

TABLE 2

Fill in the missing information.

Experiment	Events considered success	P(success)	Events considered failure	P(failure)	P(success) P(failure)
Toss a die	3	1/6	1, 2, 4, 5, or 6	5/6	1
Toss a coin	H				
Toss 2 coins	HH, TH	1/2			
Player at bat	Gets a hit				
Launch a space ship					

From your chart, answer the following:

1. P(success) + P(failure) = _____.

2. Is this always true? _____

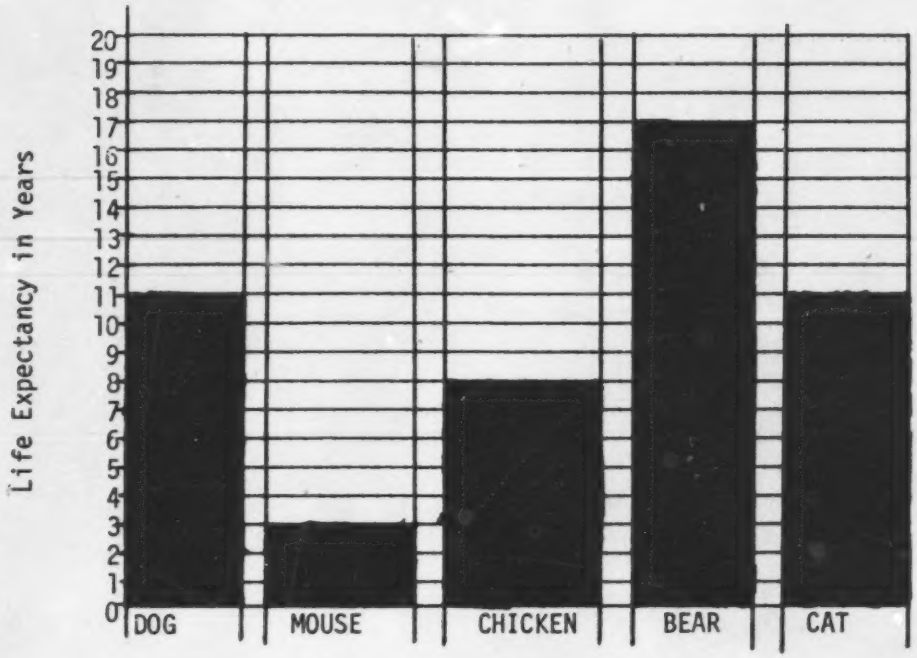
3. If the probability of getting a 5 when you roll a single die one time, is 1/6, what is the probability of <u>not</u> getting a 5? _____

4. If the probability of rain is 40% tomorrow, what are the chances that it will <u>not</u> rain? _____

5. If you have a 3 out of 4 chances of spinning "red" on a spinner, what are your chances of <u>not</u> spinning "red"?

BAR GRAPHS

Life Expectancy of Animals



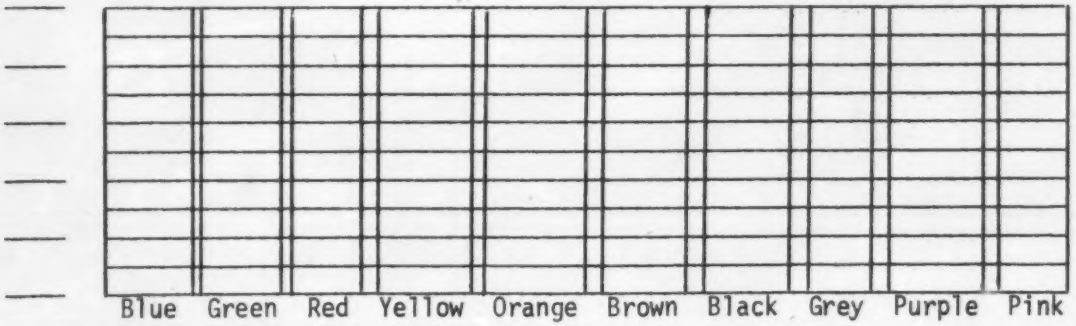
1. Which animal has the longest life expectancy? _____
2. How many years is it expected to live? _____
3. Which animal has the shortest life expectancy? _____
4. How many years is it expected to live? _____

CHOOSE YOUR FAVORITE COLOR

Cut up pieces of small colored squares. Place them in a box and have students choose their favorite color. Record the number of students in your class who selected each color on the table below.

Color	Blue	Green	Red	Yellow	Orange	Brown	Black	Grey	Purple	Pink
Number of students										

Make a graph of the number of students who selected each color. Choose a number scale for the number of students and fill in the graph.

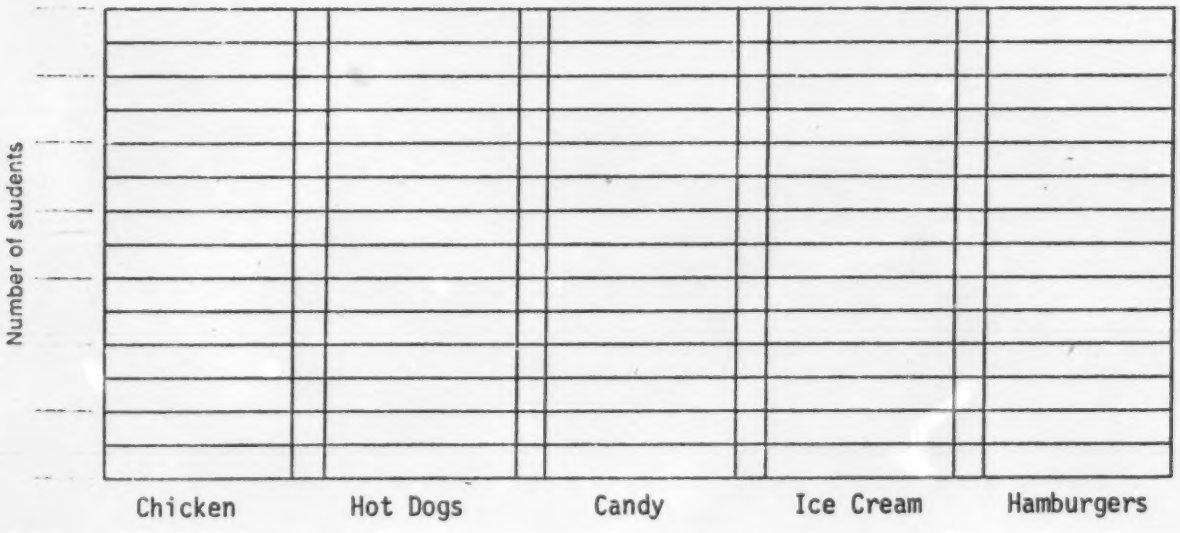


CHOOSE YOUR FAVORITE FOOD

Make a bar graph of your class's favorite food chosen from the list below. Your bar graph should show the number of students that chose each food.

Food	Chicken	Hot Dogs	Candy	Ice Cream	Hamburgers
Number of students					

BAR GRAPH

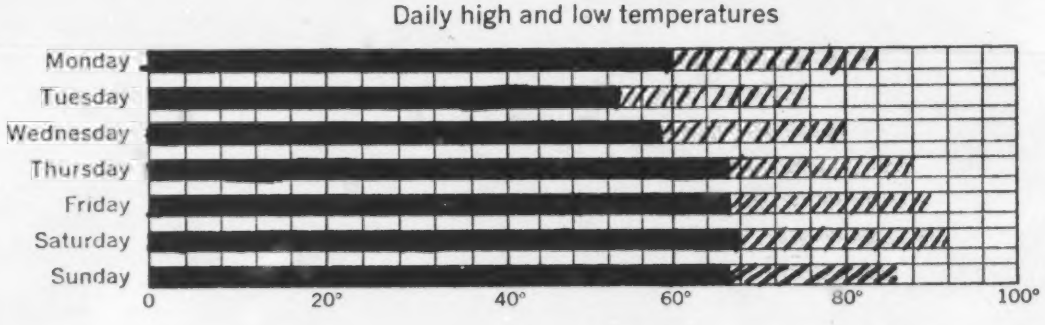


DISPLAYING DATA

In the following sections, we shall study some beginning ideas about displaying data.

The chart (or table) at the right and the bar graph below present the same information about daily high and low temperatures, but the information is presented by using different methods.

Daily High and Low temperatures		
	High	Low
Monday	84°	60°
Tuesday	76°	54°
Wednesday	80°	58°
Thursday	88°	66°
Friday	90°	67°
Saturday	92°	68°
Sunday	86°	66°



Discussion Exercises

For Discussion Exercises 1 through 7, refer to the temperature graph and chart.

1. a) On which day did the highest temperature occur?
   b) Is Part a answered more quickly by the chart or the graph?

2. a) Which day had a low temperature of 67°?
   b) Is the chart or the graph more helpful in finding the answer to Part a?

3. a) Which day did the lowest temperature occur?
   b) What was the lowest temperature of the week?

4. a) Which day shows the smallest difference between the high and low temperatures?
   b) Is the chart or the graph more helpful in finding the answer to Part a?

5. On how many days did the temperature exceed 80°?

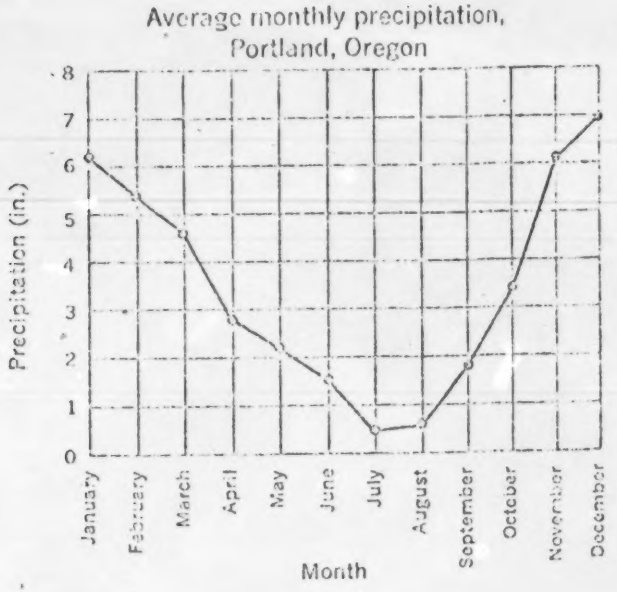
6. On which days did the temperature fall below 60°?

7. What is the difference between the highest and the lowest temperature of the week?

8. What is the range of the temperatures for the week?

LINE-SEGMENT GRAPH

Graphs like the following one are called line-segment graphs. This graph was constructed by plotting points corresponding to the month and the rainfall for that month. Then, successive points were joined by drawing line segments.



Discussion Exercises

For Discussion Exercises 1 through 4, refer to the percipitation graph.

1. Give the approximate average rainfall for each month.

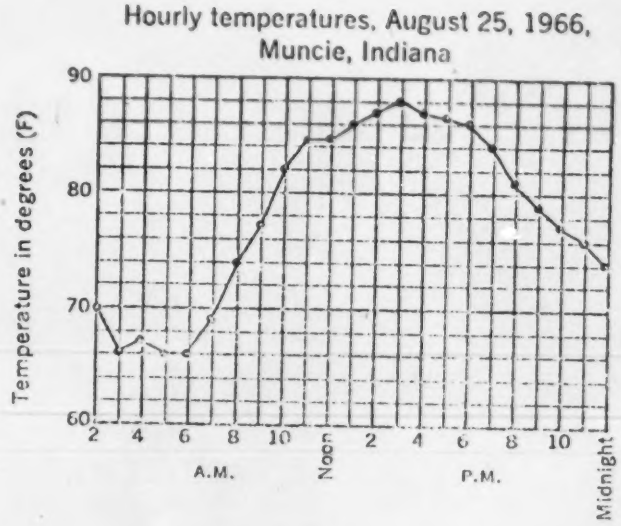
2. In which months is the average rainfall

   (a) less than 2 in.? (b) more than 5 in.?

3. Do you think the points on the segments between the months have any meaning?

4. Does the graph give you any idea of the rainfall on a particular day of the month? Explain your answer.

LINE-SEGMENT GRAPHS- 2



5. (a) What was the highest temperature on August 25? When did the highest temperature occur?

   (b) What was the lowest temperature? When did the lowest temperature occur?

   (c) How much did the temperature rise between 6:00 A.M. and noon?

   (d) How many degrees did the temperature drop between 3:00 P.M. and midnight?

6. For a decade, the average daily high temperature for the month of July is given in the table. Construct a line-segment graph to illustrate this data.

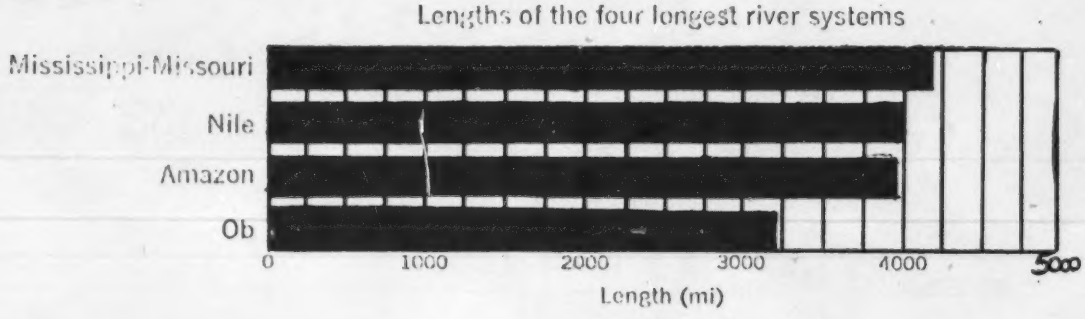
Year	Average daily high (degrees, F)	Year	Average daily high (degrees, F)
1957	84.0	1962	81.6
1958	77.4	1963	84.9
1959	83.8	1964	85.3
1960	84.8	1965	82.3
1961	83.8	1966	90.5

7. A traffic survey to determine the number of cars passing through a street intersection found the information in the table. Construct a line-segment graph to illustrate this data.

Time interval	Number of cars
6-7:00 A.M.	48
7-8:00 A.M.	244
8-9:00 A.M.	360
9-10:00 A.M.	121
10-11:00 A.M.	72
11:00 A.M. - noon	112
12-1:00 P.M.	213
1-2:00 P.M.	147
2-3:00 P.M.	139
3-4:00 P.M.	354
4-5:00 P.M.	402
5-6:00 P.M.	389

BAR GRAPH

The following horizontal bar graph compares the lengths of the four longest river systems in the world. Study the graph and note how it is constructed.



Discussion Exercies

The discussion exercies will help you to recognize and understand some of the features of horizontal bar graphs. Use the preceding graph to answer Discussion Exercies 1 through 6.

1. a) What is the title (or name) of the graph?
   b) Does the title accurately describe what the graph shows?

2. a) On the graph, 1 square represent ___?___ miles?
   b) If a scale of 1 square for each 100 miles had been used, would the bars on the graph be longer or shorter?

3. Note that all the bars are the same width and all the bars are spaced equally apart. What does this feature contribute to the graph?

4. Why is it helpful to have the rivers listed in order of their lengths?

5. a) Give the approximate length of each river shown on the graph.
   b) How accurate are your answers to Part a? Are they accurate to the nearest mile? Are they accurate to the nearest 100 miles? Explain your answers.

6. a) Would it be useful to round the lengths of each river before drawing the graph?
   b) Would you round the lengths of the rivers to the nearest 1000 miles before drawing the graph? Explain your answer.

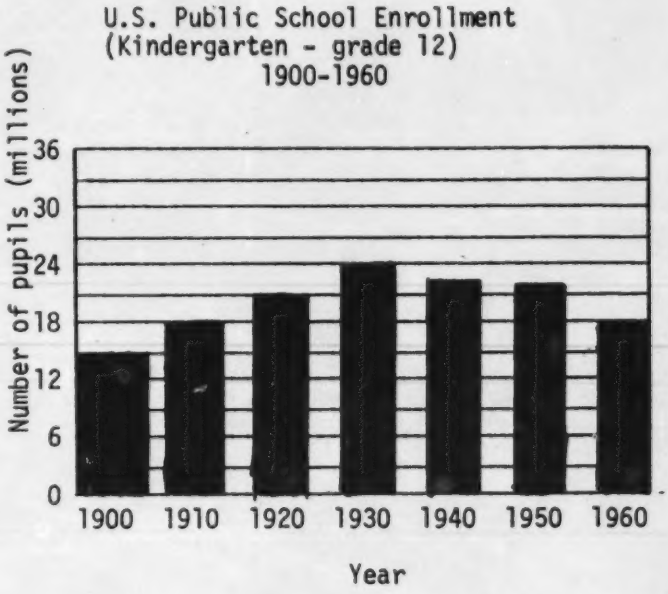
7. The vertical bar graph shows public school enrollment in the United States.

   a) Give the approximate school enrollment for each year shown on the graph.

   b) How many more pupils were enrolled in 1960 than in 1900?

   c) In which ten year period did school enrollment grow most most rapidly? What was the increase in enrollment during this period?

   d) During which years did school enrollment decrease?



8. Construct a bar graph using the heights of the five mountains. Round the height of each mountain to the nearest 1000 feet before drawing the graph.

Mountain	Height (ft)
Everest	29,141
McKinley	20,300
Popocatepetl	17,883
Matterhorn	14,705
Pikes Peak	14,110

9. Construct a vertical bar graph giving the approximate population of the United States by decades from 1900 to 1960.

Year	Approximate population (millions)
1900	76
1910	92
1920	106
1930	123
1940	132
1950	151
1960	180

10. Using the graph you constructed in Exercise 9, give an estimate of the population of United States in 1970. Use the graph to estimate the population in the year 2000.

SOME MEASURES OF CENTRAL TENDENCY

The Mean, Median and Mode

Jerry made a list of the ages of his classmates' fathers. His list comprised 25 numbers:

42,40,36,38,51,39,38,39,55,40,44,47,38,
50,34,38,43,49,42,37,41,48,52,35,45

In order to analyze the set of numbers, Jerry arranged them in order, from largest to smallest.

Rank	Age
1	55
2	52
3	51
4	50
5	49
6	48
7	47
8	45
9	44
10	43
11	42
12	42
13	41
14	40
15	40
16	39
17	39
18	38
19	38
20	38
21	38
22	37
23	36
24	35
25	34
Total	1041

The range of the set of numbers is the difference between the largest and smallest numbers in the set.

$$55 - 34 = 21$$

The median of a set of numbers is the middle number of the set.

41 is the median.

The mode of a set of numbers is the most frequently occurring number in the set, if there is such a number.

38 is the mode.

The arithmetic average or mean is the sum of all the numbers divided by the number of numbers in the set.

$$\frac{1041}{25} = 41.64$$

If the number of ages were an even number, then the median would be found by finding the average of the two "middle numbers." Notice that, in this case, the median and the mean are very close to each other; however, this is not always the case.

The mean, median, and mode are called measures of central tendency.

1. Find the range, mean, median, and mode of the list of numbers.

91,85,84,92,86,84,85,86,79,94
76,90,91,77,64,87,89,91,87,88

2. The score sheet shows the scores of a bowling team for three games.

Team Score Sheet				
Name	Game 1	Game 2	Game 3	Three-game total
Bob	136	148	128	
Chuck	158	139	145	
Lew	162	170	166	
Ike	151	144	181	
Don	169	208	195	
Team Total				

a) Find the three game total for each person on the team.
b) Find the team total for each game.
c) Find the average score of each person on the team for the three games. (Round your answers to the nearest whole number.)
d) What was the average score bowled by the team for each of the three games? (Round you answers to the nearest whole number.)

3. The scores for a mathematics test given to a class are listed below:

82,88,72,94,90,69,74,81,89,85,72,92
80,66,75,85,96,92,83,87,84,82,77,78

a) What is the range of the scores?
b) What is the median score?
c) What is the mean score?

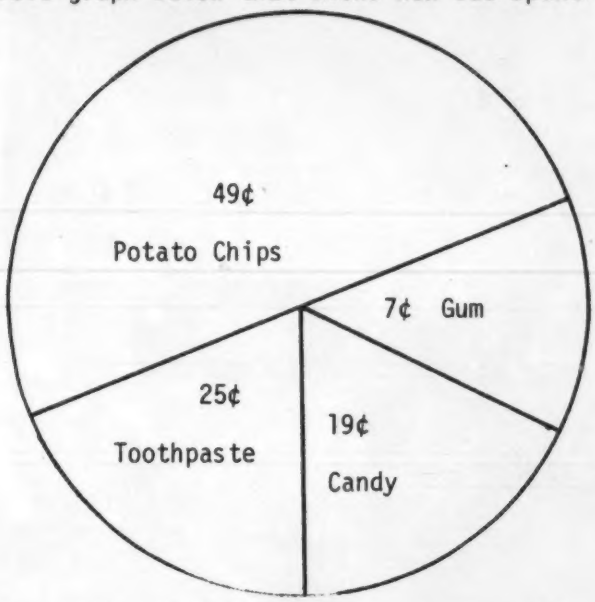
4. The weight of each starting player on a high school football team is given in the table.

   a) Compute the average weight of the starting lineup.
   b) What is the median weight of the starting lineup?
   c) What is the average weight of the starting linemen (all the players except the backs)?
   d) What is the average weight of the backfield players?

Position	Weight (lb)
Center	165
R Guard	176
L Guard	172
R Tackle	200
L Tackle	180
R End	165
L End	167
Fullback	169
R Halfback	168
L Halfback	140
Quarterback	159

CIRCLE GRAPH

A circle graph is used to show the different parts that make us a whole. Look at the circle graph below that shows how Sue spent $1.00 at the store:



1. a) What part of the $1.00 did Sue spend on candy? _____

   b) How much did she spend on gum and candy? _____

   c) On what did she spend the most money? _____

2. Make a circle graph to show how you would spend $2.00 at the store.

Answer Sheet

101. Results will vary.

102. Results will vary.

103. blue (Probability of drawing blue marble = 6/10.)

104. Results will vary.

105. a. P (3 heads) = 1/8
b. P (2 heads and 1 tail) = 3/8
c. P (1 head and 2 tails) = 3/8
d. P (3 tails) = 1/8

106. Possible outcomes: 1, 2, 3, 4, 5, or 6

107. P (staying well) = 4/6 He will probably stay well.

108. Results will vary.

109.

110. a) P (tails) = 1/2
b) Yes
c) Answers will vary. Fractions equivalent to 1/2.

111. P (picking matched pair) = 2/4

112. a) P (even number) = 6/11
b) P (odd number) = 5/11
c) P (even & multiple of 5) = 0
d) P (odd and multiple of 5) = 2/11

113. a) P (blue chip on 1st draw) = 4/12
b) P (white chip on 1st draw) = 3/12
c) P (not getting white on 1st draw) = 9/12
d) P (red on 2nd draw after red on 1st draw) = 4/11
e) There are no blue chips.

114. P (heads) = 1/2. No change in probability.

115. Results will vary.

116. P (getting an "A" card) = 14/36
P (getting an "E" card) = 2/36
"C" and "D" cards have equal chances.

Results will vary.

117. P (landing on 6) = 1/6
P (landing on 3) = 1/6
Results will vary.

118. Project will vary.

119. A is most likely to win because P (throwing 9) = 4/36, while P (throwing 11) is only 2/36.

120. Results will vary.

121. Results will vary.

122. Results will vary.

123. Results will vary.

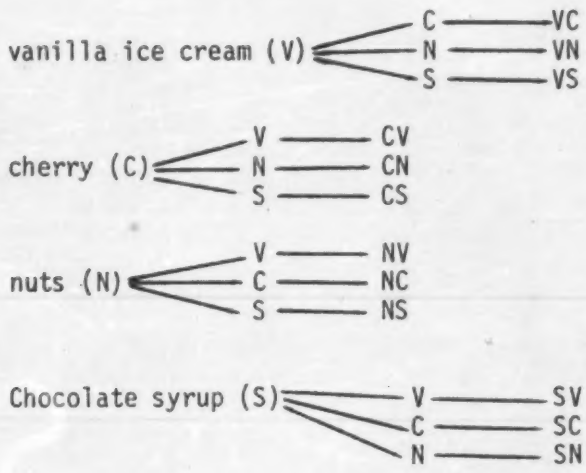
124. Results will vary.

125. 1 red, 1 yellow, or 1 red, 1 green, or 1 blue, 1 yellow, or 1 blue, 1 green (4 possible outcomes)

Answer Sheet

401. - 412. Results and graphs will vary.

413.



(Note: Two of the same ingredient have not been combined.)

414. - 415. Results and graphs will vary.

PROBABILITY

Answer Sheet

201. 1 = yes, yes, yes
2 = results will vary
3 = results will vary

202. Results will vary

203. Results will vary

204. 1. 2 possible outcomes
2. 4 possible outcomes
3. 5 possible outcomes
4. 7/30
5. 6
6. 6/30
7. 3
8. 3/30
9. 9
10. 9/30

205a. 1. answers will vary
2. table and results will vary
205b. 3. graphs will vary
205c. 4. results will varly
5. 6 possible outcomes
6. answers will vary
7. answers will vary
8. answers will vary

206. 1. HTT, THH, THT, TTH, TTT
2. 8 possible outcomes
3. 1/8
4. 1/8
5. 1 out of 8 times or 1/8 of the time
6. 1 out of 8 times or 1/8 of the time

207. 1. Results and tables will vary.
2. Results will vary.
3. Results will vary.

208a. 1. 4
2. 1/2
3. 10 times
4. 1/4

208b. 5. 5 times
6. 1/8
7. 4 times
8. 1/4
5 outcomes
1/9

208c. 9. 4 outcomes 16 times
8 outcomes 1/8 8 times
1/16 4 times

10. No, these are predictions or expected number of times.

209a. 1. 10 times
2. 14/60
3. 9/60
4. 10
5. 12
6. 6
7. 9/60
8. 60
9. 60/60

210. 1. 4 cards
2. 4 kings
3. 10/10
4. 1
5. 0/10
6. 0
7. King
8. Queen

211a. 1. Tables and results will vary.
2. Tables and results will vary.
3. 2 different outcomes: H, T

211b. 4. Answer will vary.
5. No
6. 12 times
7. 12, 11, 50
8. 12/20, 11/15; 50/100

Sums: $\frac{20}{20} = 1$, $\frac{15}{15} = 1$, $\frac{100}{100} = 1$

9. 1

211c. 10. Results and tables will vary.
11. Answers will vary
12. 2/4 or 1/2
13. 1/4

212. 1. Certain
2. equal chances
3. uncertain
4. certain
5. impossible
6. certain/equal chances
7. uncertain
8. uncertain
9. likely
10. uncertain

213a. Table 1: does not get a hit, T, HI or TH, does not take off "Failures" take all outcomes that are not success.

213b.

		$\frac{1}{2}$	T	$\frac{1}{2}$	$\frac{2}{2} = 1$
			HT, TT	$\frac{1}{2}$	$\frac{2}{2} = 1$
		$\frac{1}{2}$	does not get a hit	$\frac{1}{2}$	$\frac{2}{2} = 1$
	takes off	$\frac{1}{2}$	does not take off	$\frac{1}{2}$	$\frac{2}{2} = 1$

1. 1
2. yes
3. 5/6
4. 60%
5. 1 out of 4

213b.

	$\frac{1}{2}$	T	$\frac{1}{2}$	$\frac{2}{2} = 1$
		HT, TT	$\frac{1}{2}$	$\frac{2}{2} = 1$
	$\frac{1}{2}$	does not get a hit	$\frac{1}{2}$	$\frac{2}{2} = 1$
takes off	$\frac{1}{2}$	does not take off	$\frac{1}{2}$	$\frac{2}{2} = 1$

1. 1
2. yes
3. 5/6
4. 60%
5. 1 out of 4

GRAPHING

Answer Sheet

301. 1. Bears
2. 17 years
3. Mouse
4. 3 years

302. Results and graphs will vary.

303. Results and graphs will vary.

304a. 1.a) Saturday
b) answer will vary (graph compares)

2.a) Friday
b) answers will vary (chart shows exact temperatures, graph might be less exact to read)

3.a) Tuesday
b) 54°

4.a) Sunday
b) answers will vary (graph more pictorial - compare lengths of bars)

304b. 5.a) 5 days
6. Tuesday, Wednesday

7. 92° - 54° = 38° difference

8. 54° to 92°

305a. 1.

Jan.	6"	July	5"
Feb.	5.3"	Aug.	6"
Mar.	4.7"	Sept.	1.9"
Apr.	2.9"	Oct.	3.4"
May	2.1"	Nov.	6.1"
June	1.5"	Dec.	7"

2. a) June, July, August, September
b) January, February, November, December

3. No

4. No, it shows you the average of all the days of the month.

305b. 5. a) 88°, 3:00PM
b) 66°, 2AM, 5AM, and 6AM
c) 18° increase
d) 14° drop

6,7. Graph as illustrated by example. Note title and labels on both axes.

306a. 1. a) Lengths of the four longest river systems.
b) Yes

2. a) 250 miles
b) longer

3. It allows you to compare by sight. May be other answers discussed.

4. Easier comparison. May be other answers discussed.

5. a) Mississippi - Missouri 4,750 miles
Nile 4,000 miles
Amazon 4,200 miles
Ob 3,200 miles

b) As accurate as we can estimate between squares
(1 square = 250 miles)
Not accurate to nearest mile.
Can be accurate to nearest 100 miles.

6. a) Would not be as accurate, but depends upon what nearest figure we choose.

b) No, nearest 1000 miles not precise enough.

306b. 7. a)

1900	15,000,000
1910	18,000,000
1920	21,000,000
1930	24,000,000
1940	22,000,000
1950	21,500,000
1960	18,000,000

b) 3,000,000

c) 1900-1910, 1910-1920, 1920-1930
3 million increase

d) 1930-1960

8. & 9. Graph as illustrated by example shown.

10. Estimates must be greater than 180,000,000 as noted by trend of graph.

307a. 1. 94
92
91
91
91
90
89
88
87
87
86
86
85
85
84
84
79
77
76
64

range: 64-94
mean: 85.3
median: 86.5
mode: 91

2. a) Three game totals: 412, 442, 498, 476, 572
b) Team totals: Game 1 776
Game 2 809
Game 3 815

c) 137, 147, 166, 159, 191
d) 800

307b. 96
94
92
92
90
89
88
87
85
85
84
83
82
82
81
80
78
77
75
74
72
72
69
66

a) range: 66-96
b) median: 82.5
c) mean: 82.2

307c. 4. a) 169 lbs.
b) 168 lbs.
c) 175 lbs.
d) 159 lbs.'

308. 1. a) $\frac{19}{100}$ or 19%

b) 19¢ + 7¢ = 26¢

c) potato chips

2. Graphs will vary. Be sure parts are proportional.